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# AEOLIAN TONES OF A PLATE IN A CHANNEL

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*The conditions of the onset of aeroacoustic resonance phenomena near a plate in a gas flow in a rectangular channel are studied theoretically and experimentally in a two-dimensional formulation. The eigenfrequency as a function of the plate's chord and its position in the channel, the shape of the eigenfunctions, and the effect of the Mach number of the basic gas flow versus the eigenfrequencies and eigenfunctions and the mechanism of self-excited oscillations are determined. A mathematical model by means of which the dependence of the resonance phenomena on the geometrical parameters of the structure were performed is proposed and substantiated.*

**Introduction.** The investigation of self-excited acoustic oscillations in a rectangular channel with a plate — the simplest model of the periodic lattice of plates — is of importance, because this shape of an unbounded region is typical and occurs frequently in engineering. Self-excited oscillations that are generated in real structures are, as a rule, caused by the interaction of the eigenoscillations of an infinite region with nonlinear sources: the formation of coherent structures in a flow, unsteady combustion, etc. The important conditions of the onset of intense auto-oscillations are the coincidence of the frequency of an oscillation source with the acoustic eigenfrequency of an open infinite region and the nonorthogonality of the function that describes the source-generated acoustic waves and the eigenfunction.

The first studies of self-excited oscillations near a symmetric lattice of plates in a rectangular channel were performed by Parker [1], who showed that self-excited oscillations are purely acoustic, are not connected with plate vibrations, and are caused by coherent structures in the wake behind the plates. Usually, these oscillations are called a vortex sound or aeolian tones (Aeolus is the Greek god of the winds). Aeolian tones near a plate in a channel or near a regular grid of plates were studied theoretically and experimentally in [2–9]. In a two-dimensional formulation, self-excited vibrations near a plate in a rectangular channel in a two-dimensional formulation were treated by Sukhinin in [10]. He proposed and substantiated a mathematical model of eigenoscillations and investigated numerically the dependence of the eigenfrequencies of oscillations on the geometrical parameters of the plate in the channel.

Theoretical and experimental investigations of acoustic oscillations near a plate in a channel can be grouped as follows:

- *Studying Resonance Properties of Unbounded Media.* The existence of the acoustic resonance, the shape of self-excited oscillations (auto-oscillations), the dependence of resonant (eigen) frequencies on the geometrical parameters and the main gas flow. Mathematical simulation, numerical and experimental studies of self-excited acoustic vibrations in unbounded and semi-bounded regions.
- *Studying the Nature of a Source of Auto-Oscillations.* Formation and development of coherent structures in the wake behind a plate in laminar or turbulent flows. The interaction of the self-excited acoustic oscillations near the plate in the channel with the ordered structures in the wake behind the plate, and the effect of these structures on acoustic oscillations and the effect of oscillations on the ordered structures.

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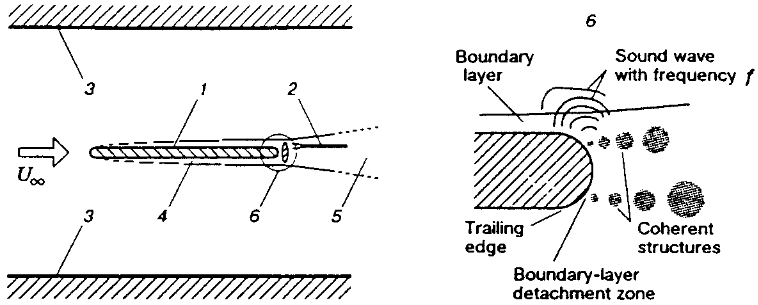


Fig. 1. Scheme of the experiment: 1) plate; 2) thermoanemometer gauge; 3) channel wall; 4) turbulent boundary layer; 5) wake; 6) generation region for coherent structures.

The Joukowski–Kutta condition at the trailing edge has not yet been clarified in constructing a rigorous mathematical model of the aeolian tones of a plate in a channel with gas flow. Most probably, the problem cannot be solved within the framework of the linear theory, because the interaction of vortex and acoustic oscillations in the neighborhood of the edge of detachment is essentially nonlinear. From the authors' viewpoint, the Joukowski–Kutta condition is not satisfied for the acoustic mode of aeolian vibrations.

In this paper, the source of aeolian tones is assumed to be ordered or coherent structures generated at the trailing edge of a plate. The possibility of control of turbulent shear flows of various types is associated precisely with the existence of these structures in the flow [11–15]. Control methods for coherent structures can be active (periodic in the time of action, for example, application of an acoustic field [13, 14]) or passive (variation of the geometry of a flow region). The phenomenon of aeroacoustic resonance, i.e., the increase in the amplitude and intensity of the aeolian tones of a plate in a channel, is an example of the implementation of flow conditions under which the presence of coherent structures and acoustic resonance properties of the main flow region are of decisive value for the flow as a whole. The acoustic and vortex modes are assumed to interact only on a plate and at the channel walls [16, 17].

**Description of the Experiment.** Measurements were performed in an MT-324 closed-type wind tunnel of the Institute of Theoretical and Applied Mechanics of the Siberian Division of the Russian Academy of Sciences whose closed operating section had a square ($0.2\times 0.2$ m) cross section of length 0.8 m. The scheme of the experiment is depicted in Fig. 1. A flat plate with rounded leading and trailing edges was located in the plane of symmetry of the working section along the flow, and the radius of rounding off was equal to half the plate thickness. The plate was positioned in the center of the working section ($H = 200$ mm). In this experiment, we used a set of plates fabricated from transparent Plexiglass of thickness 8 and 10 mm and length (chord) $L$ from 50 to 400 mm. The width was equal to the width of the working section, and it was the same for all the plates. As usual, the thickness of the half-finished sheets for plates differed by 0.1–0.3 mm from the nominal thickness of a sheet, and their surfaces were not additionally treated. To simulate a two-dimensional flow, the plates were placed such that they completely choked the working section of the channel. To examine the effect of the asymmetry of the flow region, the plate moved relative to the symmetry plane of the working section by the quantity $h = 0$–$60$ mm with a 10-mm step. The mean velocity of the incoming flow was determined using a Pitot–Prandtl tube and a micromanometer.

The mean velocity and the velocity pulsations at the local points of the flow in the wake and the boundary layer were measured by a thermoanemometric gauge with a gold-plated tungsten filament of thickness 6 $\mu$m and length 1 mm. For measurements, we used an analog thermoanemometric apparatus manufactured by DANTEC. A 55M01-type thermoanemometer had the standard 55M10 bridge with arm ratio 1 : 20, and the maximum frequency of the bridge was 200 kHz for a flow velocity around a gauge of 100 m/sec. According to the producer's data, the typical output noise level was 0.013% at a flow velocity of 10 m/sec.

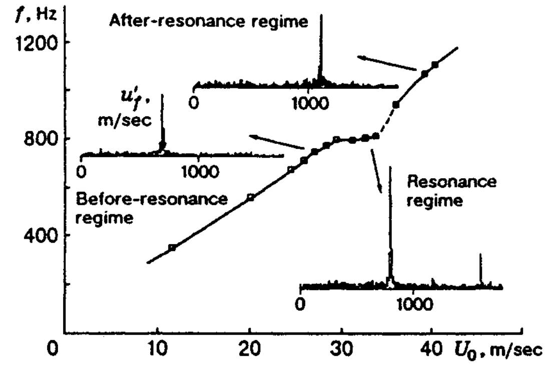


Fig. 2

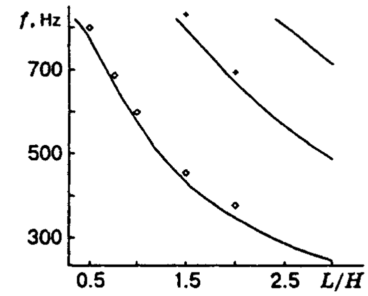


Fig. 3

Fig. 2. Spectral composition of the velocity pulsations near the trailing edge.

Fig. 3. Resonance-oscillation frequency versus the plate length: the solid curve refers to the calculation; the points ⋄, + refer to the experiment for the even and odd modes, respectively.

In pulsation measurements, the gauge moved along the plate simultaneously with the flow at various distances from its surface outside the boundary layers of the model and the channel walls. For qualitative control of the sound-pressure level, the microphone signals were used only for analysis of the spectral composition of the sound wave and as the reference signals for thermoanemometric measurements; the microphone was not calibrated. The thermoanemometer signals were processed digitally. The gauge was calibrated in a free flow near the Pitot–Prandtl tube at flow velocities ranging from 2 to 45 m/sec in such a way that the error in the determination of the mean velocity was smaller than 2%. The calibration function is described by the formula $U = k_1(E^2 - E_0^2)^{1/n} + k_2(E - E_0)^{1/2}$, where $E$ and $E_0$ are the output voltages of the anemometer bridge for the mean flow $U$ and zero velocities, and $k_1$, $k_2$, and $1/n$ are the empirically found constants. The first term corresponds to the known King expression, and the second term was added to take into account free convection. Only the longitudinal components of the mean and pulsation velocities $U$ and $u'$ were measured. These quantities were determined using a thermoanemometer from which they arrived, through a MacADIOS-Adio analog-to-digital converter manufactured by "GW Instruments," at a PC in which the signals were linearized during the experiment and were subjected, if necessary, to spectral analysis in a 4-Hz band with the use of the Fourier transform.

We used a program for collection of thermoanemometric measurement data and a number of applied programs for data recording. In particular, a library Fourier-transform subprogram was used to record the spectral characteristics. This scheme of research allowed us to perform experiments in real time. The signals arriving at the ADC were controlled by a two-beam oscillograph. The measurements were performed for velocities of the incoming flow $U_0$ ranging from 5 to 45 m/sec.

The measurements in the boundary layer of the models have shown that the turbulent boundary layer occurred on the major part of the model in the basic velocity range. The wake measurements (Fig. 2) have shown that the linear dependence of the frequency of coherent structures on the velocity of the incoming flow is violated in some velocity range, whereas the aeroacoustic resonance was observed precisely in this range.

The frequencies of the coherent structures were measured as follows. The thermoanemometer's gauge was positioned downstream at a distance from the trailing edge equal to one thickness of the plate. It is worth noting that the aeroacoustic resonance can be determined in experiments by the classical method [1]. The sound intensity in the working section of the channel increased abruptly for a certain range of the flow velocities. In this range, the fundamental frequency in the wake remained constant (the resonance regime), but the spectrum of velocity pulsations near the trailing edge contained additional harmonics (Fig. 2). The oscillation amplitude at the fundamental frequency became much larger. In the after-resonance regime, the amplitude of the fundamental frequency decreased again, and no generation of additional harmonics was observed. It is seen that the fundamental frequency of pulsations in the wake is a linear function of the flow velocity. The velocity dependence of the frequency is known to be determined by the Strouhal number, which was approximately 0.22 in these experiments. Similar results were obtained for all the models of the plates. The almost horizontal "stepwise" sections on the plane $(U_0, f)$, whose ordinates coincide with the values of the resonant frequencies, are well pronounced for most of them, and at least one resonance regime is distinctly detected. For a plate of length 300 mm, there are resonant frequencies which were less than twofold as small, and the sound level at them was comparable. For a 400-mm plate, the aeroacoustic resonance phenomena occurred as well, but the sound was much weaker at the first resonant frequency than at the second one. The experiments for plates of various lengths made it possible to determine the dependences of the resonant frequencies on the lengths of the models. Figure 3 shows numerical and experimental data on the resonant frequencies versus the length of the plate's profile (relative to the channel height).

Thermoanemometric measurements were carried out to obtain data on the frequency and shape of resonant acoustic oscillations. The gauge scanned the entire surface of the model and the areas beyond its leading and trailing edges for each fixed distance from the plate along the $Y$ coordinate. The measurements were performed in the region lying outside the boundary layers of the plate and the walls of the channel's working section. The spectral analysis was carried out at each point, and the sound-wave frequency and the oscillation amplitude were measured at this frequency. As a result, we obtained data on the distribution of the amplitude of velocity pulsations in a standing sound resonant-frequency wave in the space between the plate and the channel wall along the $X$ and $Y$ coordinates.

**Calculation and Comparison with Experiment.** The geometry of the main flow and the region of acoustic oscillations are shown in Fig. 1. It is known [16] that the solution of the equations of gas motion, which are linearized with respect to the main flow, can be expanded into the sum of vortex and acoustic modes. In the case considered, this statement is not true for the entire flow region (the expansion is violated at the trailing edge). However, one can assume that the expansion into vortex and acoustic modes holds in the entire flow region except for the trailing edge, the unknown singularity of the solution at the trailing edge is described by the vortex mode (this is in agreement with Howe's viewpoint [17]), the acoustic oscillations are caused only by the vortex mode, and the effect of acoustic waves on the sound source should be taken into account only in the regimes in which the acoustic resonance phenomena (the increase in the amplitude) appear.

Acoustic and vortex oscillations generated by the vortex detachment can be considered steady-state in time in the coordinate system related to the plate. This means that some ordered vortex structure that determines the frequency of acoustic oscillations is fixed in the wake. The potential $u(x, y, t)$ of the acoustic velocity perturbation is of the form $u(x, y, t) = u(x, y) \exp(i\omega t)$.

According to this, the equations for the potential of the acoustic velocity perturbation, which describe the acoustic oscillations $u(x, y)$ of a steady-state gas flow, take the form

$$(1 - \mathrm{M}^2) u_{xx} + u_{yy} - \frac{2i\omega \mathrm{M}}{c} u_x + \frac{\omega^2}{c^2} u = 0 \quad \text{in } \Omega, \tag{1}$$

where $\mathrm{M} = U/c$ is the Mach number of the main flow, $c$ is the velocity of sound, $\omega$ is the circumferential frequency of acoustic and vortex oscillations in the plate's reference system, and $i$ is an imaginary unity. Equation (1) holds in the entire flow region $\Omega$. The Neumann conditions for an immobile plate, which are true for the sum of acoustic and vortex modes, should be satisfied at the channel wall $B$ and the plate profile $\Gamma$. The vortex mode $v$, which is determined by the vortex structures in the wake, is regarded as a known function of coordinates. The acoustic and vortex modes at the channel walls and the plate are related by the relation

$$u_y = -v(x, y) \quad \text{on } \Gamma + B. \tag{2}$$

The form of Eq. (2) can be simplified if the solution is searched for in the form

$$u(x,y) = \bar{u}(x,y)\exp\left[i\frac{\omega}{c}\frac{\mathrm{M}x}{(1-\mathrm{M}^2)}\right] \tag{3}$$

and the dimensionless variables $\xi = x/(H\sqrt{1-\mathrm{M}^2})$ and $\zeta = y/H$, where $H$ is the channel height (see Fig. 1), are introduced. In dimensionless variables, for the unknown function $\bar{u}(\xi,\zeta)$, Eq. (1) has the form

$$u_{\xi\xi} + u_{\zeta\zeta} + \lambda^2 u = 0 \quad \text{in } \Omega. \tag{4}$$

Here and below, $\lambda = H\omega/(c\sqrt{1-\mathrm{M}^2})$ has the sense of a dimensionless frequency and the bar above the function $u$ is omitted everywhere. For a more convenient comparison with experimental results, we can use the expression $\lambda = (2fH\pi)/(c\sqrt{1-\mathrm{M}^2})$, where $\omega = 2\pi f$ and $f$ is the oscillation frequency measured in Hertz. In the new variables, the width of the channel is equal to unity, and to the plate length $L$ corresponds the dimensionless quantity $l = L/(H\sqrt{1-\mathrm{M}^2})$, which characterizes the length of the plate profile relative to the channel height with the kinematic correction because of the flow. The correction increases the real length of the profile.

It is noteworthy that if the Neumann condition (2) is satisfied for $u(x,y)$, the function $u(\xi,\zeta)$ is subject to a similar condition in which the required replacement of the variables was performed:

$$u_\zeta(\xi,\zeta) = -v(\xi,\zeta) \quad \text{on } B+\Gamma. \tag{5}$$

Relations (4) and (5) describe the forced oscillations near the plate in the channel. They are linear, and, therefore, one can search for the solution in the form $u = u + w$, where $w$ is the partial solution of Eq. (4) subject to the inhomogeneous boundary conditions (5), and $u$ is the general solution of this equation subject to the homogeneous boundary conditions $u_\zeta(\xi,\zeta) = 0$ at $B+\Gamma$.

Problem (4) and (6) describes self-excited vibrations near the plate in the channel. Problem (1) and (2) reduces to problem (4) and (5) by replacing the variables.

**Remark.** After the replacement of the variables, the study of aeroacoustic resonance phenomena (aeolian tones) near a plate in a channel in a gas flow is equivalent to the study of eigenoscillations near a plate in a channel without a gas flow.

• *Dependence of the Frequency on the Length of the Plate Profile.* The eigenfrequency of oscillations versus the profile length is shown in Fig. 3. Good agreement of the experimental and theoretical results allows us to draw a conclusion on the high accuracy of mathematical simulation of self-excited oscillations. For short lengths of the profile, the difference between the theoretical and experimental data can be explained by the strong effect of the plate thickness because, in this case, it becomes comparable with the length of the plate's profile. We note that the dimensionless frequency tends to $\pi$ at $L \to 0$.

• *Dependence of the Frequency of Self-Excited Vibrations on the Position of the Plate in the Channel.* Figure 4 shows experimental data on the first resonant frequency versus the position of the plate. Bardakhanov and Poroshin [18, 19] have found that the resonant acoustic frequency varied in displacing the plate from the symmetry plane. In the present paper, the dependence of the resonant frequency on the position of the plate in the channel was derived. Measurements were performed as follows. A plate of length 150 mm was displaced at some distance $h$, which is a multiple of 10 mm, to the upper wall of the working section. The distance between the plate plane and the upper wall was halved, and the thermoanemometer's sensor moved over the model, measuring the frequency and amplitude of pulsations in the sound wave. The plate was shifted to the next position $h$, and the process was repeated. As a result, the resonant frequencies, the maximum values of the amplitudes, and the wave shape were obtained for each value of $h$. Figure 4 shows the dependence of the first resonant frequency of oscillations on the position of the plate, which is in agreement with the results of [10]. Clearly, the displacement of the plate from the symmetry axis leads to an increase in the resonant frequency. The corresponding points are well approximated by a quadratic parabola. The sound-wave amplitude measurements have shown that the vibrations are localized in the region between the plate and the wall that is the closest to it.

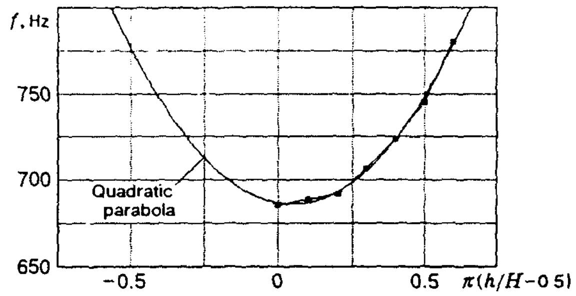


Fig. 4

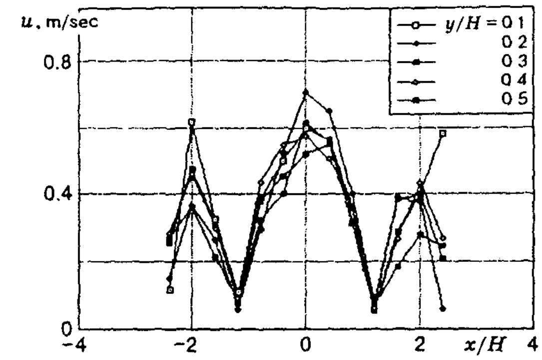


Fig. 5

Fig. 4. The first resonant frequency versus the position of the plate ($L = 150$ mm) in the channel of height $H = 200$.

Fig. 5. Acoustic oscillations of the velocity on the longitudinal coordinate.

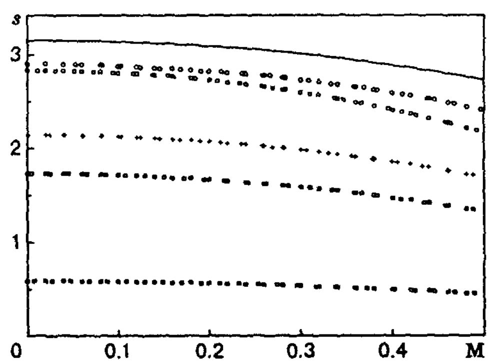


**Fig. 6. The reduced frequency of self-excited oscillations $s$ versus the main-flow Mach number for the length of the plate profile $L = 0.5$, 1, and 5 (points o, +, and □, respectively).**

The calculated data of [10] on the dependence of the first eigenvalues for the even and odd modes for a fixed profile length on the position of the plate in the channel are in agreement with the experimental data presented in Fig. 4.

**Form of Eigenfunctions and Mechanics of Self-Excited Oscillations.** Figure 5 illustrates the measurement of the velocity pulsations in the sound wave above a model of length 400 mm (the measurement procedure was described above). The measurement data are given for the second resonant frequency, which was equal to 692 Hz for this plate. The distributions for the first resonant frequency are different from those shown in Fig. 5: they have two maxima located in the region near the leading and trailing edges of the plate, rather than three maxima. Recalculation permits one to derive a formula for the sound wave from these distributions.

The effect of the basic gas flow on the shape of the eigenfunctions (aeolian tones) is due to in the fact that they are related to the eigenfunctions of problem (1) and (2) by the transformation (deformation) (3).

In addition, we have to note that the influence of the Mach number on the frequency of self-excited oscillations near the plate in the channel (Doppler effect) are taken into account using the relations for the dimensionless frequency $\lambda = \omega H/(c\sqrt{1 - M^2})$ and length $l = L/(H\sqrt{1 - M^2})$. Figure 6 shows the dependence of the first reduced frequencies of self-excited oscillations near a plate with a fixed length of the chord on the gas-flow Mach number in the channel. The quantity $s = \lambda\sqrt{1 - M^2} = \omega H/c$ is called a reduced eigenfrequency.

We note that the frequency of self-excited oscillations decreases with increasing flow velocity.

**Conclusion.** (1) A mathematical model that describes the aeolian tones of a plate in a channel has been proposed and verified experimentally.

(2) The dependence of the frequency of self-excited oscillations on the length of the plate profile and the plate's position in the channel has been found numerically and experimentally.

(3) The form of the self-excited oscillations for the first modes has been found experimentally and numerically.

(4) The effect of the main-flow velocity on the frequency of self-excited oscillations (Doppler effect) has been studied.

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